Seat No. : _______________

N15-108

November-2014

B.Sc., Sem.-V

MAT-303 : Mathematics

(Complex Analysis and Fourier Series)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.

(2) Each question carry **14** marks.

1. (a) State and prove De Moivre's theorem. **7**

OR

If z_1 and z_2 are complex numbers then prove that

(i) $|z_1 + z_2| \leq |z_1| + z_2|$

(ii) $|z_1 - z_2| \geq |z_1| - z_2|$

(b) If $\sin(\alpha + i\beta) = x + iy$, then prove that **7**

(i) $\dfrac{x^2}{\cosh^2\beta} + \dfrac{y^2}{\cosh^2\beta} = 1$

(ii) $\dfrac{x^2}{\sin^2\alpha} - \dfrac{y^2}{\cos^2\alpha} = 1$

OR

For what values of z does the series $\sum (-1)^n (z^n + z^{n+1})$ converge ? Also find its sum.

2. (a) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Also express f(z) in terms of z. **7**

OR

Prove that the necessary conditions for a function $f(z) = u + iv$ to be analytic at all points in a region R are

(i) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

(ii) $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

(b) Prove that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin. **7**

OR

Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of (x, y) but are not harmonic conjugates.

3. (a) If f(z) is analytic and $f'(z) \neq 0$ at each point z of the domain then prove that the mapping $w = f(z)$ is conformal. **7**

OR

Find the image of strip $\frac{1}{4} < y < \frac{1}{2}$ under the mapping $w = \frac{1}{z}$.

(b) Considering the map $w = z\, e^{i\frac{\pi}{4}}$, determine the region R′ of w – plane corresponding to the triangular region R bounded by the lines $x = 0$, $y = 0$, $x + y = 1$ in z – plane. **7**

OR

Show that the transformation $w = \frac{2z + 3}{z - 4}$ transforms the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$ and explain why the curve obtained is not a circle.

4. (a) Find the Fourier series expansion for the function $f(x) = x^2$ in $[-\pi, \pi]$. Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots..$ **7**

OR

Find the Fourier series expansion for the function $f(x) = x + x^2$ in $[-\pi, \pi]$.

(b) State and prove Bessel's inequality of the Fourier series. **7**

OR

If $\phi(x)$ is Riemann integrable in $a \leq x \leq b$ and if $A_n = \int_a^b \phi(x) \cos nx \, dx$ and

$B_n = \int_a^b \phi(x) \sin nx \, dx$, then prove that $A_n \to 0$, $B_n \to 0$ as $n \to \infty$.

5. Answer the following. **14**

(1) Find the modulus and principal argument of the complex number $-\sqrt{3} - i$.

(2) Find the square root of the complex number $3 - 4i$.

(3) Write C-R equations in polar form.

(4) Is the function $f(z) = \frac{1}{z}$ analytic ? If yes, then find its derivative.

(5) Transform the curve $x^2 - y^2 = 4$ under the mapping $w = z^2$.

(6) Define conformal and isogonal transformations.

(7) The Fourier series of the even function contains only cosine terms and that of odd function contains only sine terms. Is it true ? Why ?
